PATIENT'S INFLUX AT OUTPATIENT MEDICAL LABORATORY (OPML);

FORECASTING OF PATIENT'S INFLUX AT MAYO HOSPITAL, LAHORE USING ARIMA MODEL

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ABSTRACT... Objective: Uni-variate time series data analysis has been reported in medical literature for prediction the patients volume, emergency overcrowding, stay length in hospital on surgical procedure and bed occupancy in hospital wards, patients influx or patients arrival, moreover to estimate the cost of hospital stay or any medical or surgical procedure. **Design:** The present study was designed to fit an appropriate uni-variate ARIMA model (Box-Jenkin methodology) to forecast the patient's incoming at OPML, Mayo Hospital, Lahore. **Setting:** Mayo Hospital Lahore, **Period:** September 2009 to December 2013 were used for fitting the best model. **Method:** Time series data of male, female and peads patients coming/reporting in OPML, OPD. **Result & conclusion:** The appropriate model for male and female data was found as ARIMA (1, 0, 1) and for peads as ARIMA (1, 1, 1) after residuals diagnostic checks. The estimated number of male patients for Month of January2014 is 632 whereas the actual incoming of male patients in the month of January, 2014 was 649, which shows that the estimated model has ability to forecast the number of incoming patients accurately. It is now concluded that the fitted ARIMA model can be used to forecast the patients incoming to OPD Medical Laboratory for future planning and management.

Key words: Forecast, Medical Time Series Data, ARIMA Model, Box-Jenkin Methodology

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INTRODUCTION

Hospital Outpatients Medical Laboratory (OPML) overcrowding has become a severe problem throughout Pakistan as case around the world, which consequents in increasing care costs, causing stress in Medical Laboratory Professionals (MLP's), patients and their attendants and also affecting adversely the outcomes of medical & clinical systems. One side of problem is the difficulty of anticipating the timing and magnitude of overcrowded condition.¹ Modeling and forecasting the patients volume using some most appropriate statistical, may provide useful information for hospital administration and Medical Laboratory Professionals (MLP's), which may be useful in planning, expansion, allocating resources and recruitment.^{1,2,3,4}

Uni-variate time series data analysis has been reported in medical literature for prediction of the patients volume, emergency overcrowding, stay length in hospital on surgical procedure and bed occupancy in hospital wards, patients influx or patient arrival, moreover to estimate the cost of hospital stay or any medical or surgical procedure.^{5,6,7}

Several authors used time series data analysis techniques like AR (Autoregressive) MA (Moving Average) ARIMA (Autoregressive Integrated Moving Average) model in forecasting the patient's arrival behavior and trend in hospitals. Time series Models and techniques provide better results than traditional approaches such as descriptive statistics, multiple regression and ANOVA.^{8,9,10}

OBJECTIVE OF STUDY

The health/patients related data, which is collected over a sequence of time at regular interval normally shows a non-stationary behavior and clinical researchers are often interested to forecast the future occurrence of such related phenomena. The ARIMA model has the ability

to handle the non-stationary time series data to forecast. The objective of this study is to forecast the incoming of patients at OPML, OPD, Mayo Hospital Lahore.

ARIMA MODEL

The most popular model for forecasting of univariate time series data is ARIMA model, which was introduced by Box-Jenkin in 1976.^{11,12} As many researchers used this model for forecasting of different medical phenomenon, in paper we have used ARIMA model for forecasting of future incoming of patients at OPML Mayo Hospital Lahore. The ARIMA model is combination of two time series processes Autoregressive (AR) and Moving Average (MA). The ARIMA model is defined as

$$X_{t} = \Phi_{1}X_{t-1} + \Phi_{2}X_{t-2} + \dots + \Phi_{p}X_{t-p} + Z_{t} + \theta_{1}Z_{t-1} + \theta_{2}Z_{t-2} + \dots + Z_{q}\theta_{t-q}$$
(1)

The ARIMA model is used for integrated stationary process and is best when time series is non-stationary, the non-seasonal ARIMA is classified as ARIMA(p,d,q)¹³⁻¹⁶, the process X, is said to be ARIMA

If
$$\nabla^d X_t = (1-B)^d X_t$$
 (2)
 $W_t = \alpha_t W_{t,1} + \alpha_2 W_{t,2} + \dots + \alpha_t W_{t,1} + Z_t + \theta_t Z_{t,1} + \dots + \theta_t Z_t$

We can say, the process is stationary after differencing the process "d" times, normally in practice the "d" is taken one. The ARIMA model building procedure which is also known as Box-Jenkin method, involves the following four steps:-

- 1. Identification of Model
- 2. Estimation of model parameters
- 3. Diagnostic checks (model checking)
- 4. Forecasting

The flow chart of the model is as below:





IDENTIFICATION OF MODEL

In this step, the order of respective time series model is determined, the order of differencing "d", the order of AR process which is "p" and order of MA process is "q". The model is written as ARIMA (p,d,q). Differencing is the procedure which is used to remove the trend in the time series data, we use differencing to remove the trend in the data. It is critical to difference the time series data, until stationary is getting done. Any seasonal variation can be removed through some appropriate seasonal differencing. The stationarity of time series means that arithmetic mean, variance and autocorrelation remain same over time. It means that if time series data is stationary, then the arithmetic mean of any major subset of data of time series data does not significantly different of any other major subset of the series.^{17,18}

Value of "d"

The order of differencing "d" is estimated through autocorrelation plot, when the autocorrelations die out quickly, then the appropriate value of "d" can be found. Most statistical forecasting methods are based on the assumption that the time series can be turned into approximately stationary using of some sort of mathematical transformations. The first difference of a time series is the series of changes from one period to the next. If X(t) denotes the value of the time series X at period t, then the first difference of X at period t is equal to X_m -X_{m-1}

Value of 'p"

The order of AR (Autoregressive) process "p" is determined from PACF (partial autocorrelation function) of the differenced series. If the PACF cut off after few lags (a period of time between one event and another), the last lag with the larger value would be the estimated value of "p".

Value of 'q"

The order of MA (moving average) process is determined from ACF (auto correlation function) of differenced time series.

PROCEDURE FOR CHECKING THE STATIONARITY

The time plot, correlogram and unit root test are used to test the stationarity of the time series data. The time plot steers us about the trend of data, whereas correlogram guides us regarding the order of ARIMA model and behavior of the series. The unit root test is performed to determine the significance of stationarity in the data. There are several available statistical methods for unit rot test of non-seasonal time series data, those tests are Dickey-Fuller (DF), Augmented Dickey-Fuller (ADF, 1979)Zivot (1992), and KwaitKowski (1992). For a given data set, there may be several model with different orders, for example ARIMA (1, 1, 1), ARIMA (2, 1, 1) and more, but final model is selected using statistical tools like AIC (**Akaike information criterion**) and BIC(**Bayesian information criterion**) **or Schwarz criterion**. The AIC and BIC are tools to measure the goodness of fit of model. Among number of models, that model is considered best which has lower value of AIC or BIC.¹⁸⁻²⁵

ESTIMATION OF PARAMETERS

Next step after the tentative ARIMA model is the estimation of parameters. Least square or maximum likelihood method can be used to estimate the parameters. The parameters are estimated on the basis of previously available data.

DIAGNOSTIC CHECK

The tentative suggested ARIMA model is then checked for reasonable fit to the data or we can say that in this stage, we verify the statistical adequacy of the fitted model. Statistically an adequate model must satisfy the four properties:-

- i) The estimated parameters must be significant
- ii) All Auto Regression (AR) estimated parameters must be within the "bound of statisonarity". the condition of bound stationarity is as:-

AR order	Condition of bound
AR(1)	$ \Phi_1 < 1$
	$ \Phi_2 < 1$
AR(2)	$(\Phi_2 - \Phi_1) < 1$
	$(\Phi_2 + \Phi_1) < 1$

- iii) All Moving Average (MA) estimated parameters must lie in the bound of stationarity.
- iv) The Residuals of model (the unexplained part of data) should be least and R² should be maximum; the possible test for minimum residuals, Q-statistic is introduced by Ljung-Box (Ljung and Box 1979, Harry 1993).

FORECASTING

The eventual target of ARIMA model is to forecast. When the model has undergone all the diagnostic check, the model becomes adequate for forecasting. Various researches showed that ARIMA model performs lustrous in term of forecasting as compared to other models.

DATA ACQUISITION AND ANALYSIS STRATEGY

The data of monthly patient's records at OPML OPD Mayo Hospital, Lahore from September 2007 to December 2013 was acquired for modeling. Computer software E-view 5 (Quantitative Micro Software (QMS), USA) and R (Freeware software)²⁶ was used to analyze the time series data. The following tools were used for fitting the model.

Step	Statistical Tools	Purpose
Identification	Time Plot Correlogram	To see the trend in data To show the relationship among current and past observations
	Unit Root Test	To check the stationarity in the data
Estimation	Maximum Likelihood estimation (MLE)	To estimate the parameters
Diagnostic Check	R ² , Goodness of fit AIC, BIC	To find out the most appropriate model
Forecasting and its Accuracy	RMSE(Root Mean Squared Error), MAE(Mean Absolute Error), MAPE(Mean Absolute Percentage Error),	To forecast and check the accuracy in model using forecast error term

RESULTS

The original time series data of male, female and peads patients were plotted to observe the behavior or trend in the data, plotting of data is considered a first step to identify the time series model. The line charts of data are shown in Fig 1-4.



Fig-1. The time plot of all data



Fig-2. The time plot of male data



Fig 3. The time plot of female data



Fig 4. The time plot of peads data

STATIONARITY CHECKING

Autocorrelation function (ACF) is a widespread tool to check the stationarity of the time series data. ACF & PACF are also used to propose the order of the tentative model. The correlogram (graph of ACF & PACF) of male and female data is stationary at zero difference (d=0) whereas the pediatrics data is adjusted for stationarity at d=1, shown in Fig 5-8.

Apart from the graphic methods, the ADF unit

	Correlogram	OT MALE					
Date: 03/13/14 Time: 21:56 Sample: 2007M09 2013M12 Included observations: 76							
Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob		
		$\begin{array}{cccccccccccccccccccccccccccccccccccc$	-0.008 0.080 0.128 -0.012 -0.082 -0.0012 -0.097 -0.051 0.066 0.033 0.145 -0.033 -0.109 -0.133 -0.109 -0.133 -0.060 -0.048 0.065 0.107	5.3310 13.002 16.636 17.213 17.245 20.313 20.916 21.170 21.214 22.379 22.868 23.263 23.297 23.406 23.583 23.297 23.406 24.146 24.479 25.156 25.462 24.146 24.479 25.156 25.653 27.907 28.668 28.964 29.9133 29.673 29.673 29.673	0.021 0.002 0.002 0.004 0.008 0.005 0.007 0.012 0.020 0.034 0.050 0.034 0.050 0.034 0.050 0.062 0.034 0.175 0.213 0.261 0.298 0.340 0.378 0.397 0.437 0.482 0.436 0.430 0.467 0.554 0.584		

Fig 5. The Correlogram of Male data

	Correlogram of F	EMA	LE				
Date: 03/13/14 Time: 21:59 Sample: 2007M09 2013M12 Included observations: 76							
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob	
		$\begin{array}{c} 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \\ 22 \\ 23 \\ 24 \\ 25 \\ 26 \\ 27 \\ 28 \\ 29 \\ 30 \\ 31 \end{array}$		0.082 -0.090 0.137 0.097 0.218 0.074 -0.214 0.054 -0.066 0.014 0.027 -0.134 -0.042 -0.050 0.033 -0.029 0.003 -0.029 0.003 -0.029 0.003 -0.029 0.003 -0.029 0.003 -0.029 0.003 -0.029 0.003 -0.029 -0.050 -0.050 -0.050 -0.050 -0.050 -0.050 -0.055 -0.0	8.7678 14.600 15.386 15.656 16.024 16.109 16.754 17.640 17.656 19.752 22.931 23.198 23.198 23.691 23.198 23.691 23.198 23.691 24.147 26.714 27.901 30.879 31.545 31.703 32.344 23.826 33.827 34.159 34.507 34.507 34.509 34.693	0.003 0.001 0.002 0.004 0.007 0.013 0.019 0.026 0.049 0.026 0.049 0.028 0.039 0.050 0.050 0.050 0.050 0.072 0.085 0.103 0.072 0.085 0.106 0.072 0.085 0.112 0.072 0.085 0.112 0.072 0.085 0.116 0.016 0.119 0.112 0.139 0.161 0.1221 0.221 0.228 0.300 0.344	

Fig 6. The Correlogram of Female data

rig of the conclegium of remaie data						
	Correlogram	of P	EADS			
Date: 03/14/14 Tim Sample: 2007M09 2 Included observation	013M12					
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		$\begin{array}{c} 10\\ 11\\ 12\\ 13\\ 14\\ 15\\ 16\\ 17\\ 18\\ 19\\ 20\\ 21\\ 22\\ 23\\ 24\\ 25\\ 26\\ 27\\ 28\\ 29\\ 30\\ 31\\ \end{array}$	0.623 0.475 0.335 0.297 0.288 0.175 0.068 -0.003 -0.094 -0.122 -0.101 -0.108 -0.089 -0.078 -0.091 -0.053 -0.066 -0.079 -0.105 -0.109 -0.114 -0.127 -0.137 -0.137 -0.135 -0.185	-0.051 0.008 0.091 -0.123 -0.060 -0.050 -0.004 -0.017 -0.047 -0.029 0.025 0.057	$\begin{array}{c} 124.41\\ 127.04\\ 127.44\\ 127.44\\ 128.23\\ 129.59\\ 130.54\\ 132.41\\ 133.00\\ 133.81\\ 134.17\\ 134.45\\ 134.91\\ 135.57\\ 136.64\\ 138.19\\ 139.64\\ 138.19\\ 139.64\\ 138.57\\ 136.64\\ 138.19\\ 139.64\\ 141.41\\ 143.58\\ 145.20\\ 147.47\\ 150.03\\ 152.41\\ 156.03\\ 152.41\\ 156.03\\ 160.88\\ \end{array}$	0.000 0.0000 0.000 0.000 0.000 0.00000 0.0000 0.0000 0.000000
Fig 7. The Correlogram of Peads data					lata	

Fig 7. The Correlogram of Peads data

	Correlogram of D(P	EADS)					
Date: 03/14/14 Time: 20:35 Sample: 2007M09 2013M12 Included observations: 75							
Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob		
		7 -0.055 8 -0.103 9 0.096 10 -0.156 11 -0.122 12 0.125 13 -0.068 14 0.028 15 0.058 16 -0.118 17 0.107 18 0.044 19 -0.033 20 0.033 21 -0.007 22 -0.094	-0.100 0.008 -0.285 -0.129 0.189 -0.050 -0.171 0.024 -0.067 -0.149 -0.038 -0.064 0.011 -0.092 -0.113 0.144 -0.026 -0.040 -0.040 -0.050 -0.050 -0.050 -0.057 0.025 -0.069 0.007 0.064 -0.037	19.780 20.210 20.285 20.614 21.976 23.109 23.301 23.411 23.527 23.532 24.491 24.936 24.936 24.936 24.936 24.936 24.931 27.482 27.812 28.119 30.294	0.597 0.603 0.729 0.173 0.046 0.070 0.081 0.096 0.074 0.074 0.071 0.150 0.144 0.179 0.220 0.244 0.316 0.322 0.354 0.322 0.433 0.4470 0.322 0.433 0.4470 0.52500 0.52500 0.5250000000000		
Fig 8. The Correlogram of Peads data at d=1							



root test (also a very popular method to check the stationairty of the data) was also performed to confirm the stationarity of the data shown in table-I.

Variable	ADF test statistic (p value) d=0	ADF test statistic (p value) d=1			
Male	-6.80(0.000)	-9.7589 (0.000)			
Female	-6.52 (0.000)	-14.07 (0.000)			
Peads	-2.85 (0.06)	-8.97 (0.000)			
Table-I. ADF test results					

MODEL SPECIFICATION

The ACF & PACF of male, female and Peads data is shown in Fig 5-8. The correlogram of male and female data indicated that the order of model is not more than 2 lags and order of peads data is not more than 3 lags. So the tentative/proposed model may be ARIMA(1,0,0), ARIMA(1,0,1), ARIMA(1,1,1), ARIMA(2,0,1), ARIMA(2,1,1). All parameters of the models are estimated using Maximum Likelihood Estimation (MLE) method. The diagnostic checks were also done using Ljung & Box methods. In addition to that, the best model was selected on the following criteria with maximum R² and minimum AIC/BIC, RMSE, MAE and MAPE.

MAXIMUM LIKELIHOOD ESTIMATION OF PARAMETERS

The summary of estimated parameters is in table-II for male, table-III for females and table-IV for peads data.

	Proposed models for male data				
Parameters	ARIMA (1,0,0)	ARIMA(1,0,1)	ARIMA(1,1,1)		
С	625.51	640.88	-0.456		
AR	0.259	0.734	0.209		
MA		-0.567	-0.99		

Table-II. Summary of estimated parameters of proposed model

	Proposed models for female data				
Parameters	ARIMA (1,0,0)	ARIMA (1,0,1)	ARIMA (1,1,1)		
С	897.17	919.84	-0.734		
AR	0.33	0.746	0.308		
МА		-0.567	-0.997		

Table-III. Summary of estimated parameters of proposed model

	Proposed models for Peads data					
Parameters	ARIMA (1,0,0)	ARIMA (1,0,1)	ARIMA (1,1,1)			
С	106.92	107.91	-2.422			
AR	0.82	0.78	0.455			
МА		0.075	-0.107			
Table-IV. Summery of estimated parameters of proposed model						

3.4 DIAGNOSTIC CHECK

The summary of diagnostic tools of all proposed models are shown in Table-V for males and best model is selected as ARIMA (1, 0, 1), for female in Table-VI and best model is ARIMA(1,0,1) and for Peads in Table-VII. 7, and best model is

ARIMA(1,1,1)

	Proposed models for male data				
Parameters	ARIMA (1,0,0)	ARIMA (1,0,1)	ARIMA (1,1,1)		
R ²	0.072	0.170	0.040		
BIC	13.07	13.02	13.12		
AIC	13.01	12.93	13.02		
RMSE	161.04	154.33	203.19		
MAPE	22.65	21.09	97.23		
MAE	117.86	112.04	139.67		
Table-V. the summary of diagnostic check for male					

	Proposed models for female data			
Parameters	ARIMA (1,0,0)	ARIMA (1,0,1)	ARIMA (1,1,1)	
R ²	0.166	0.169	0.036	
BIC	13.93	13.93	13.98	
AIC	13.87	13.83	13.89	
RMSE	253.38	242.54	301.78	
MAPE	30.54	29.21	100.014	
MAE	197.90	190.92	232.54	
Table-VI. The summary of diagnostic check for female data				

	Proposed models for peads data			
Parameters	ARIMA (1,0,0)	ARIMA (1,0,1)	ARIMA (1,1,1)	
R ²	0.564	0.655	0.70	
BIC	10.89	10.95	11.00	
AIC	10.83	10.86	10.81	
RMSE	85.34	85.95	54.48	
MAPE	151.54	152.37	107.61	
MAE	61.33	61.67	36.22	
Table-VII. The summary of diagnostic check for peadsdata				

The fitted ARIMA (1, 0, 1) for male data can be described as:

$$\begin{split} X_{t} &= \phi_{1}X_{t-1} + Z_{t} + \theta_{1}Z_{t-1} \\ X_{t} &= 640.8 + 0.734X_{t-1} - 0.567Z_{t-1} \\ \end{split}$$
(3) The fitted ARIMA (1, 0, 1) for female data can be described as:

 $\begin{aligned} X_t &= \phi_1 X_{t-1} + Z_t + \theta_1 Z_{t-1} \\ X_t &= 919.\$ + 0.746 X_{t-1} - 0.566 Z_{t-1} \end{aligned}$

$$W_{t} = \phi_{1}W_{t-1} + Z_{t} + \theta_{1}Z_{t-1}$$

$$W_{t} = -2.422 + 0.455W_{t-1} - 0.107Z_{t-1}$$
(5)

FORECASTING

Three months forecasting of male, female and peads patient's is estimated on the basis of best selected model is shown in table-VIII.

	Male		Female		Peads	
Month	Esti- mated	Actual	Esti- mated	Actual	Esti- mated	Actual
Jan 2014	632	649	889	874	119	102
Feb 2014	626	599	886	812	118	98
March 2014	623	675	883	980	117	110
Table–VIII. The estimated and actual number of patients						

CONCLUSION

A best model for Male, Female, and Peads patient's data is developed for forecasting on the basis of available tools. The empirical analysis indicated that ARIMA(1,0,1) is best fitted model for male and female patients data and ARIMA(1,1,1) for peads for short run forecasting. The estimated number of patients for next three months is given in Table No. 8 and compare with actual data.

The finding of this study may be useful for future planning, clinical researchers, hospital administration, and Government also for the welfare and betterment of patients and health care staff.

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ETHICAL CONSIDERATION

This study was carried out on the patient's data available at OPDCL records and there was

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no direct interaction with patient, no patients personal identification was used, so the study has not reviewed by Institutional Review Board (IRB). Copyright© 17 April, 2015.

REFERENCES

- Arul E, Mark I.C, Donald N, Leo Y.S. Using autoregressive integrated moving average (ARIMA) models to predict and monitor the number of beds occupied during a SARS outbreak in a tertiary hospital in Singapore. BMC Health Services Research 2005;(5):36.
- Champion R , Kinsman L , Lee G , et al., Forecasting emergency department presentations, Aust Health Rev 2007; 31(1): 83-90.
- 3. Cameron P, Hospital overcrowding: a threat to patient safety? Med J Aust 2006;184(5):203-4.
- Basu K, Gupta A: A physician demand and supply forecast model for Nova Scotia. Cah Sociol Demogr Med 2005; 45(2–3):255–285.
- Imhoff M., Bauer M., Gather U., Löhlein D. Time series analysis in intensive care medicine. Appl Cardio Patho.1997; (6): 263-281.
- Hwang U, Richardson L.D, Sonuyi T.O, Morrison R.S, "The effect of emergency department crowding on the management of pain in older adults with hip fracture," Journal of the American Geriatrics Society 2006; 54(2): 270–5.
- Touloumi G, Pocock SJ, Katsouyanni K, Trichopoulos D. Short-term effects of air pollution on daily mortality in Athens: a time-series analysis. Int J Epidemiol 1994; 23: 957-67.
- Akhtar S, Rozi S. An autoregressive integrated moving average model for short-term prediction of hepatitis C virus seropositivity among male volunteer blood donors in Karachi, Pakistan. World J Gastroenterol 2009; 15(13): 1607-1612.
- Berman J.J, Meyer J, Coats G: Effects of program characteristics on treatment utcome: an interrupted time-series analysis. J Stud Alcohol 1984; 45: 405-10.
- 10. Lambert C.R, Raymenants E, Pepine C.J. Timeseries analysis of long-term ambulatory myocardial ischemia: effects of beta-adrenergic and calcium channel blockade. Am Heart J 1995; 129: 677-84.
- BOX G.E.R. JENKINS G.M. Time Series Analysis, Forecasting and Control, Holden Day, San Francisco 1976.

- 12. Box G.E.P, Jenkins G.M, Reinsel G.C: **Time Series Analysis: Forecasting and Control 3rd edition.** Englewood Cliffs, NJ: Prentice-Hall; 1994
- 13. Hamilton J.D: **Time Series Analysis Princeton:** Princeton University Press; 1994.
- 14. Cooper R.A, Getzen T.E, McKee H.J, Prakash L: Economic and demographic trends signal an impending physician shortage. Health Aff 2002, 21(1):140–153.
- Santric M. M, Matejic B, Terzic S.Z, Vasic V: The emigration intentions among medical students from Serbia. Eur J Public Health 2012; 22(suppl 2):174.
- Forster A.J, Stiell I, Wells G, et al. The effect of hospital occupancy on emergency department length of stay and patient disposition. Acad Emerg Med 2003;10:127e33.
- Pankratz, Forecasting with Univariate Box-Jenkins Models: Concepts and Cases, Wiley, New York, NY, 1983.
- Hoot N.R, Zhou C, Jones I, et al. Measuring and forecasting emergency department crowding in real time. Ann Emerg Med 2007;49:747e55.
- 19. Akaike H. **On the likelihood of a time series model.** Statistician 1978; 27: 217-35.
- 20. Schwarz G. Estimating the dimension of a model. Ann Statist 1978; 6: 461-4.
- Dickey D.A, Fuller W.A. Distribution of estimators for autoregressive time series with a unit root. J. Am. Stat. Assoc. 1979;74, 427–431.
- 22. Perron, P. **Trends and random walks in macroeconomic time series: further evidence from a newapproach.** J. Economic Dynamics and Control 1988; 12, 297–332.
- Zivot, Eric , Andrews, Donald W K. "Further Evidence on the Great Crash, the Oil-Price Shock, and the Unit-Root Hypothesis," Journal of Business & Economic Statistics, American Statistical Association 1992; 10(3): 251-70.
- Kwiatkowski D, Peter C.B.P, Peter S and Yongcheol S. Testing the Null Hypothesis of Stationarity against the Alternative of a Unit Root: How Sure Are We That Economic Time Series Have a Unit Root? Journal of Econometrics 1992; 54(1-3): 159-178.
- 25. Ljung G.M, Box G.E.P. On a measure of lack of fit in time series models. Biometrika 1978; 65: 297-303.
- 26. Cryer, J .D. and Chan, K. Time Series Analysis: with

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